

# Frequency dependent power and energy flux density equations of the electromagnetic wave



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## ABSTRACT

The calculation of the power and energy of the electromagnetic wave is important for numerous applications. There are some equations to compute the power and energy density of the electromagnetic wave radiation. For instance, the Poynting vector is frequently used to calculate the power density. However those including the Poynting vector are not perfect to represent the actual values because the equations are frequency independent. In the present study we have derived the frequency-dependent equations to calculate the power and energy flux density of the electromagnetic wave by help of the classical electromagnetic theories. It is seems that the Poynting vector with a certain electric and magnetic fields is correct only for a specific frequency. However our equations are perfect to calculate the values of the power and energy flux density for all frequencies of the electromagnetic radiation. The equations may help to develop the applications of the electromagnetic wave radiation.

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## Introduction

The electromagnetic (EM) wave has been used in many applications such as telecommunications, photovoltaic, light related sensor etc. To calculate the power and energy of EM wave the appropriate equations are needed. There are some power and energy equations of the EM wave in the society. James M. Hill et al. [1] has derived an equation for the power density of the microwave, in which the power density is measured with the unit  $W/m^3$ . The power density equation has been given as;

$$Q = \frac{1}{2} \omega \varepsilon' E^2 (W/m^3) \quad (i)$$

where  $\varepsilon' = \frac{\sigma}{\omega}$ ,  $\sigma$  is the electric conductivity,  $\omega$  is the angular frequency and  $E$  is the electric field intensity. It seems that the power density is not depended on frequency after putting the value of  $\varepsilon'$  into the Eq. (i). The Poynting vector [2,3] has been used as the power density of the EM wave with the unit of  $W/m^2$ , however it is frequency independent. Tokumar [4] has derived an equation for the energy density, which is frequency independent as well. The equation is;

$$W_c = \frac{\delta(\omega\sqrt{\varepsilon\mu})}{\delta\omega} \frac{\vec{E} \cdot \vec{H}^*}{2} (J/m^3) \quad (ii)$$

where  $\varepsilon$  is the electric permittivity,  $\mu$  is the magnetic permeability and  $H^*$  is the complex conjugate of the magnetic field. Wolski [5] has stated an energy density equation of EM wave. The equation has been written as;

$$U = \frac{1}{2} \varepsilon_0 \vec{E}_0^2 (J/m^3) \quad (iii)$$

Here  $\varepsilon_0$  is the electric permittivity in vacuum and  $\vec{E}_0$  is the maximum value of the periodic electric field. The equation is frequency independent. Therefore, the EM waves with the same amplitudes of the electric field  $\vec{E}$  and magnetic field  $\vec{H}$  have the same energy regardless of their frequencies. However, we believe that two different EM waves with the same amplitude of the electric and magnetic fields but with different frequencies have to have two energies because the source as well as the wave exerts more work in a time in case of higher frequency. So, the frequency independent equations are not able to calculate the actual power and the energy of EM wave radiation. Wolski [5] has derived also a power equation that is frequency dependent. However, the current amplitude ( $I_0$ ) and the length of current ( $l$ ) are involved in this equation. The wave vector  $k$  ( $= \frac{2\pi f}{c}$ ) is the frequency part of the equation.  $C$  is the speed and  $f$  is the frequency of the EM wave. The equation is;

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$$P = \frac{(I_0 l)^2 k^2}{12\pi\epsilon_0 C} \text{ (W)} \tag{iv}$$

Based on this equation, the power is not measurable from EM radiation. In the present study, we have derived the equations of the power and energy flux density of the EM wave by help of the classical electromagnetic theories [2,5,6]. In these equations the frequency, the electric field intensity, the magnetic field intensity, the electric permittivity and magnetic permeability are associated. The units of the power and energy flux density in the equations are  $\text{W/m}^2$  and  $\text{J/m}^2$ , respectively. The power and energy flux density are measurable perfectly using our equations after knowing the values of the EM wave frequency and some other parameters. We hope that these equations could help to develop the applications of the EM wave radiation.

**Theory**

The LC circuit is usually used as a source of EM wave. The frequency of the EM wave is of the frequency of the source. The LC circuit has been discussed very briefly in the sub sections then the power and energy flux density equations of the EM wave have been derived based on it.

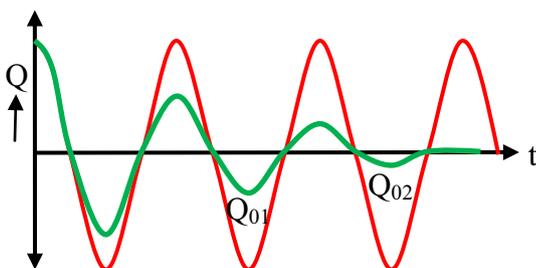
*LC circuit without bias*

Let us consider that an LC circuit with a negligible ohmic resistance is activated without any power source. The second order differential equation of charge  $Q$  or current  $I$  can be derived easily [7] with respect to time  $t$ . The stored magnetic [8] and electric energy have been exchanged periodically between the capacitor and inductor in the circuit. In this way the system radiates the EM wave with the frequency  $f$  ( $\omega = 2\pi f$ ) around the circuit. Considering the initial current is zero and the capacitor is fully charged ( $Q_0$ ) and thus the generated electric field  $\vec{E}(\vec{r}, t)$  and magnetic field  $\vec{H}(\vec{r}, t)$  have been varied (during the oscillation) at a point with following relations;

$$\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r}) \cos \omega t \tag{1}$$

$$\text{and, } \vec{H}(\vec{r}, t) = \vec{H}_0(\vec{r}) \sin \omega t \tag{2}$$

Here the phase difference is  $\pi/2$  in between the electric and magnetic field, and  $\vec{E}_0(\vec{r})$  and  $\vec{H}_0(\vec{r})$  are the maximum values of the electric and magnetic fields nearby of the circuit, respectively. Here  $\vec{r}$  is the distance of the considering point from the source of the EM wave. However the LC circuit loses the energy by the EM radiation. The fields  $\vec{E}_0(\vec{r})$  and  $\vec{H}_0(\vec{r})$  are decreased with the time at this biasing condition. Thus, the amplitude of the charge  $Q_0$  of the capacitor and the amplitude of the current  $I_0$  of the inductor are decreased grad-



**Fig. 1.** Transient response of the charges with time for an LC circuit; without power supply after charging by a DC (green line) and with AC power supply (red line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

ually, as shown in Fig. 1 (green line). By considering an infinitesimal amount of charge  $\delta Q$  that has been lost during an infinitesimal time  $\delta t$  in a form of EM radiation under an instant voltage  $V$ , the power loss  $P_s$  of the device can be written as;

$$P_s = -V \frac{\delta Q}{\delta t} \text{ (W)} \tag{3}$$

The minus sign indicates that the charge has been decreased with time.

*LC circuit with periodic bias*

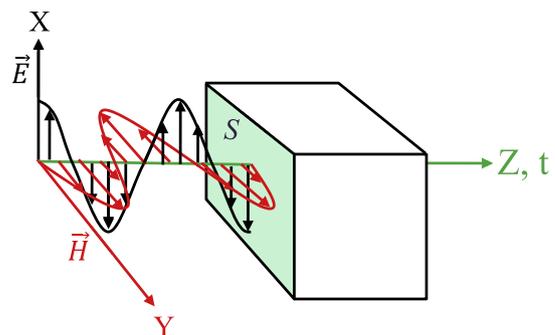
When an AC source is connected in series with LC circuit, the dissipated power ( $P_s$ ) in the LC circuit could be recovered by the power supply. The amplitude of the charge  $Q_0$  of the capacitor (and the amplitude of the current  $I_0$  of the inductor) remains constant as shown in Fig. 1 (red line). Thus the values of the fields  $\vec{E}_0(\vec{r})$  and  $\vec{H}_0(\vec{r})$  remain constant with time at a place with the resonance frequency ( $\omega$ ) of AC source and the LC circuit. Accordingly, the lost power in the form of the radiation under this biasing condition can be written as;

$$P_s = IV \text{ (W)} \tag{4}$$

where the current  $I$  ( $= -\frac{\delta Q}{\delta t}$ ) is the lost current due to the radiation, not the actual current of the device.

*The power and energy flux density of the EM wave*

Let us consider a monochromatic EM wave propagates three dimensionally across a surface  $S$  in a medium (Fig. 2) whose electric permittivity and magnetic permeability are  $\epsilon$  and  $\mu$ , respectively. To avoid any complexity, the EM wave is plane polarized and thus the electric field is perpendicular to the magnetic field. The EM wave applies an oscillating Lorentz force on a free charge located in the medium. The motion of the free charge depends on the phase difference between the electric and the magnetic fields. Here the phase difference is  $\pi/2$  as noted in the sub Section 2.1. In accordance the free charge moves on a circular orbit in the  $xz$ -plane; based on the vector rule of Lorentz force. The plane is made by the direction of the electric ( $x$ ) field  $E$  and the direction of propagation ( $z$ ) of the EM wave. The oscillation of the charge produces the two dimensional alternating current  $I(\vec{r}, t)$  and an emf  $V(\vec{r}, t)$  in the medium. However the motive of the induced resultant power  $P$  (current and emf) is directed toward the outer direction from the source. This is because the EM wave power ( $P_z$  as well as  $P$ ) has been propagated from source toward the outside ( $z$  direction). Other part (oriented in the  $E$  direction) is minimized by the interaction of the positive (one side) power and the negative (opposite side) power (i.e.  $P_x = -P_x$ ) at a point in the space. Accord-



**Fig. 2.** Plane polarized EM wave flows through a surface  $S$  in a medium.

ingly the resultant transmitted power ( $P$ ) of the EM wave could be calculated based on the classical equation;

$$P = I'(\vec{r}, t)V'(\vec{r}, t)(W) \tag{5}$$

The power  $P$  flows through the surface  $S$  which is a part of the supplied power  $P_s$  (Eq. (4)) of the source, so it is smaller than the source power, so;  $P < P_s$ .

Since the  $V'(\vec{r}, t)$  of Eq. (5) is the induced emf by the EM radiation then according to the Faraday's laws the equation can be written as;

$$P = \int_S \vec{J}(\vec{r}, t) \cdot \delta\vec{S} \left( -\frac{\delta\vec{\varphi}(\vec{r}, t)}{\delta t} \right) \tag{6}$$

where  $\vec{J}(\vec{r}, t)$  is the current density and  $\delta\vec{S}$  is the small surface area which become the total surface area ( $S$ ) of a certain space by integration.  $\frac{\delta\vec{\varphi}(\vec{r}, t)}{\delta t}$  is the rate of change of the magnetic flux  $\vec{\varphi}(\vec{r}, t)$  with respect to time. The minus sign comes from Lenz's correction of Faraday's law. The power density (intensity of the EM radiation) on the surface  $S$  can be written as:

$$P_D = \frac{P}{S} (W/m^2) \tag{7}$$

Since  $\vec{\varphi}(\vec{r}, t) = \vec{B}(\vec{r}, t)S$  and  $\vec{J}(\vec{r}, t) = \frac{\delta\vec{D}(\vec{r}, t)}{\delta t}$ , where  $\vec{B}(\vec{r}, t)$  is the magnetic induction and  $\vec{D}(\vec{r}, t)$  is the charge displacement, then from Eqs. (6) and (7) we can write the power density;

$$P_D = \frac{P}{S} = - \int_S \frac{\delta\vec{D}(\vec{r}, t)}{\delta t} \cdot \delta\vec{S} \left( \frac{\delta\vec{B}(\vec{r}, t)}{\delta t} \right) \tag{8}$$

It is well known that  $\vec{D}(\vec{r}, t) = \epsilon\vec{E}(\vec{r}, t)$ ,  $\vec{B}(\vec{r}, t) = \mu\vec{H}(\vec{r}, t)$  [2]. The power density is main concern; it should not be considered separately into the current density and induced voltage in the right side of the equation. The quantity of the left hand side of the above equation is scalar and it has been propagated from source to the outer side. For instance, at a point of the surface  $S$  (Fig. 2) the direction of the power is left to right thus, the direction of the vector cross product of the fields  $\vec{E}(\vec{r}, t)$  and  $\vec{H}(\vec{r}, t)$  is left to right, and this resultant direction of the vector cross product should be the dot product with the surface vector (which is directed also from left to right) to be the scalar quantity to the right side of the Eq. (8). Therefore, the equation could be written as:

$$P_D = -\mu\epsilon \int_S \frac{\delta\vec{E}(\vec{r}, t)}{\delta t} \times \frac{\delta\vec{H}(\vec{r}, t)}{\delta t} \cdot \delta\vec{S} \tag{9}$$

Putting the values of  $\vec{E}(\vec{r}, t)$  and  $\vec{H}(\vec{r}, t)$  from Eqs. (1) and (2) into Eq. (9) we get,

$$P_D = -\mu\epsilon \int_S \frac{\delta}{\delta t} \{ \vec{E}_0(\vec{r}) \cos \omega t \} \times \frac{\delta}{\delta t} \{ \vec{H}_0(\vec{r}) \sin \omega t \} \cdot \delta\vec{S}$$

After differentiation, the above equation could be written as;

$$P_D = \mu\epsilon\omega^2 \int_S (\vec{E}_0(\vec{r}) \cos \omega t \times \vec{H}_0(\vec{r}) \sin \omega t) \cdot \delta\vec{S}$$

$$\text{or, } P_D = \mu\epsilon\omega^2 \int_S (\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)) \cdot \delta\vec{S}$$

According to the wave nature of the electromagnetic fields we can state the EM wave velocity,  $C = \frac{1}{\sqrt{\epsilon_0\mu_0}}$  or,  $\frac{\omega^2\lambda^2}{4\pi^2} = \frac{1}{\epsilon\mu}$  and the wave vector,  $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$ . Here  $\lambda$  is wavelength of the EM wave. Consequently, the power density equation could be written as:

$$P_D = k^2 \int_S (\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)) \cdot \delta\vec{S} \tag{10}$$

This is the power density equation that is associated with frequency and the Poynting vector. The equation reveals that the power density is proportional to the square value of the frequency for the constant Poynting vector at a place. The reported power equation (iv) is frequency dependent with square value, however the Poynting vector as well as the electric field and the magnetic field are absent in that equation.

The amplitude of the propagating electric and magnetic fields attenuates with distance from the source (considering the electric field only for avoiding the complexity of the three dimensions) as shown in Fig. 3. Let us consider that an infinitesimal plane surface  $\delta\vec{S}(\vec{r})$  is a part of the surface  $S$  which is placed in an EM wave with an angle  $\theta$  between the propagation direction and the direction of the surface as shown in Fig. 3a. The plane  $\delta\vec{S}(\vec{r})$  is perpendicular to the plane of the page. So, the reader does not can to see the area of the surface but edge. The direction of the plane is shown by a dashed arrow in the figure. The EM wave touches the left end of the surface  $\delta\vec{S}(\vec{r})$  at the position  $z_1$  firstly and then at the position  $z_2$ . The amplitudes of the electric and magnetic fields are comparatively smaller at the position  $z_2$  than the position  $z_1$ . Accordingly, the resultant of the  $\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)$  varies with respect to the surface  $\delta\vec{S}(\vec{r})$  of the Eq. (10). If the EM wave falls perpendicularly on the surface  $S$ , then the angle  $\theta$  between the propagation direction and the direction of surface is  $0^\circ$ . Then the amplitudes of the electric and magnetic fields are same over the whole surface of  $\delta\vec{S}(\vec{r})$  as well as  $S$  at the place as shown in Fig. 3b. The direction of the resultant of the cross product  $\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)$  is along to the propagation direction. For this case the  $\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)$  or the modulus of the  $\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)$  will be the independent with surface  $\delta\vec{S}(\vec{r})$  and then the Eq. (10) can be written as;

$$P_D = \frac{4\pi^2 f^2}{C^2} |\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)| \int_0^S \delta S$$

Here considering  $k = \frac{2\pi f}{c}$ , and the quantity  $\int_0^S \delta S$  should be equal to  $S$ , the above equation will be as;

$$P_D = \frac{4\pi^2 f^2 S}{C^2} |\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)| \tag{11}$$

The Eq. (11) shows that the power density of EM wave depends not only on the quantity of  $\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)$  but also on squared value of the frequency of it. A graph has been plotted with the power density  $P_D$  versus the frequency  $f$  according to the Eq. (11) in which the value of the surface has been taken  $S = 1 \text{ m}^2$  and  $|\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)| = 4 \text{ W/m}^2$  as shown in Fig. 4. The graph reveals that the Poynting vector is true for the frequency  $1.33 \times 10^7 \text{ Hz}$  only in this arrangement. The power density is smaller than the Poynting vector for the case of smaller of that frequency and bigger for the case of bigger of that frequency. That indicates that the Poynting vector is not perfect to calculate the power density for all frequency at a place. The relation between the energy and the power is:

$$P = \frac{\delta E}{\delta t} (W)$$

$$\text{or, } E = \int P \delta t \text{ (J)} \tag{12}$$

By help of the Eqs. (7) and (12), we get the energy flux density on the place as;

$$E_D = \frac{E}{S} = \int P_D \delta t \text{ (J/m}^2) \tag{13}$$

From Eqs. (10) and (13) we can write;

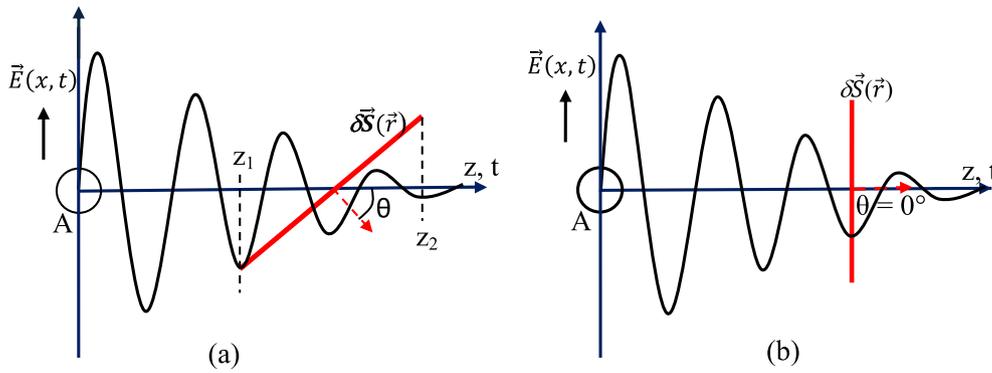


Fig. 3. The EM wave (considering electric field only) on a small surface  $\delta\vec{S}(\vec{r})$  (a) with an angle  $\theta$  between the propagation direction and the direction of the surface  $\delta\vec{S}(\vec{r})$  (b) with the value of the angle  $\theta = 0^\circ$ .

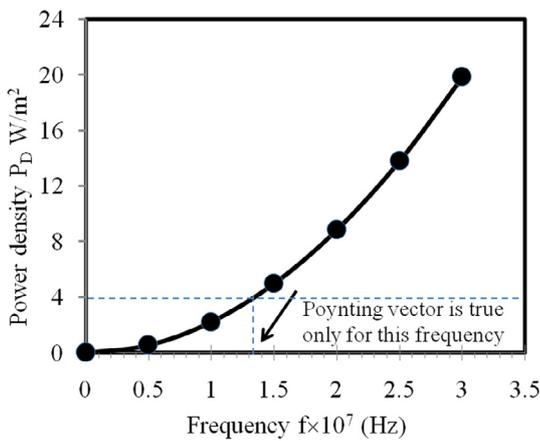


Fig. 4. The power density of the EM wave with source frequency. Here considering the value of  $S$  is  $1 \text{ m}^2$  and  $|\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t)|$  is  $4 \text{ W/m}^2$ .

the energy flux density will be zero frequently after a time period. Let us consider when  $t = 0$ , then  $E_D = 0$ . So,  $A = \frac{k^2 S}{4\omega} E_o(r) H_o(r)$ . Putting this value in Eq. (16), we can get the equation as;

$$E_D = \frac{k^2 S}{4\omega} E_o(r) H_o(r) (1 - \cos 2\omega t)$$

$$\text{Or, } E_D = \frac{1}{2} \pi f \mu \epsilon S E_o(r) H_o(r) (1 - \cos 2\omega t) \tag{17}$$

Here  $k = \frac{2\pi f}{c}$  and  $C = \frac{1}{\sqrt{\epsilon\mu}}$ . The energy flux density is proportional to the frequency with unit  $\text{J/m}^2$ , and the reported energy flux density Eqs. (ii) and (iii) are frequency independent with unit  $\text{J/m}^3$ . We have plotted the  $E_D$  with time  $t$  according to the Eq. (17) by taking  $\frac{1}{2} \pi f \mu \epsilon S E_o(r) H_o(r) = 1 \text{ (J/m}^2\text{)}$  as shown in Fig. 5. The graph shows that the nature of  $E_D$  is sinusoidal. We should take the average value for a cycle to getting a continuous and smooth value of  $E_D$ , so we get;

$$E_{DAve} = \frac{1}{2} \pi f \mu \epsilon S E_o(r) H_o(r) \int_0^T \frac{1}{T} (1 - \cos 2\omega t) dt$$

$$\text{Or, } E_{DAve} = \frac{1}{2} \pi f \mu \epsilon S E_o(r) H_o(r) \tag{18}$$

Here  $T$  is the time period of the electric and magnetic fields. The  $\vec{E}_o(\vec{r})$  and  $\vec{H}_o(\vec{r})$  are constants at a given position but varying with distance from the source. In the similar way under the stated condition of the EM wave radiation we can find the average power density  $P_{DAve}$  from Eq. (11) as;

$$E_D = \int \left\{ k^2 \int_S \vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) \delta\vec{S} \right\} \delta t$$

The value of surface is time independent.

$$\text{so, } E_D = k^2 \int_S \left[ \int \{ \vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) \} \delta t \right] \cdot \delta\vec{S} \tag{14}$$

This is the energy flux density equation, which is associated also with the frequency and the Poynting vector. To make simple the above equation, according to the considered angle between the electric and magnetic fields ( $\pi/2$ ) the Eq. (14) could be written as:

$$E_D = k^2 \int_S \left[ \int \{ E(r, t) H(r, t) \} \delta t \right] \delta S$$

From Eqs. (1) and (2), and considering the EM wave falls perpendicularly on the surface  $S$ , the above equation can be written as;

$$E_D = k^2 \left[ \int \{ E_o(r) \cos \omega t \cdot H_o(r) \sin \omega t \} \delta t \right] \int_0^S \delta S$$

$$\text{or, } E_D = \frac{k^2 S}{2} E_o(r) H_o(r) \int (\sin 2\omega t) \delta t \tag{15}$$

By integrating the above equation;

$$E_D = -\frac{k^2 S}{4\omega} E_o(r) H_o(r) \cos 2\omega t + A \tag{16}$$

Here  $A$  is the integral constant. The Eq. (16) indicates that the energy flux density is sinusoidal at a place. So it is possible that

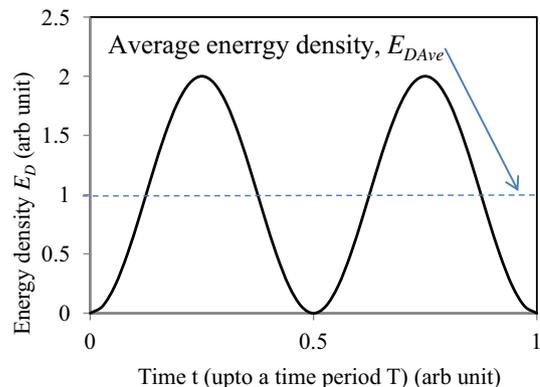


Fig. 5. The energy flux density of the EM wave with time for a frequency with the saying value  $\frac{1}{2} \pi f \mu \epsilon S E_o(r) H_o(r) = 1 \text{ (J/m}^2\text{)}$ .

$$P_{DAve} = 2\pi^2 f^2 \mu \epsilon S E_o(r) H_o(r) \frac{1}{T/4} \int_0^{T/4} \sin 2\omega t \cdot dt$$

$$\text{Or, } P_{DAve} = 4\pi f^2 \mu \epsilon S E_o(\mathbf{r}) H_o(\mathbf{r}) \quad (19)$$

The power density, average power density, energy flux density and average energy flux density of the EM wave radiation have been derived. The equations are capable to calculate the actual power and energy flux density of the EM wave.

### Conclusion

Based on the classical electromagnetic theories, the frequency dependent power density, average power density, energy flux density and average energy flux density equations of the EM wave have been derived. In these equations the frequency and the Poynting vector are associated. We have plotted the power density versus frequency graph according to our equation. The graph reveals that the Poynting vector is correct only for a frequency in an arrangement. Accordingly, we believe that our equations are perfect to calculate the actual values of the power and energy flux density of the EM radiation. We hope that it will be helpful to develop the applications of the electromagnetic wave radiation.

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